Sequence

Definition

Presentation

Pattern ... What is the next term?

1\. $\{1, 2, 4, 8, 16, \ldots\}$

2\. $\{12, -6, 3, -1.5, 0.75, \ldots\}$

6\. $\{2, 5, 10, 17, 28, 41, \ldots\}$

5\. $\{1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\}$

4\. $\{1, 2, 4, 7, 11, 16, 22, 29, \ldots\}$

3\. $\{3, 10, 17, 24, 31, 38, \ldots\}$

Functional

$f : \mathbb{N} \to \mathbb{R}$

$f(n) = \frac{n}{n+1}$ $\quad \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\}$

$f(n) = \frac{1 + (-1)^{n+1}}{2}$ $\quad \{1, 0, 1, 0, \ldots\}$

$a_n = \frac{n}{n+1}$ $\quad \left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$

$a_n = \frac{1 + (-1)^{n+1}}{2}$ $\quad \left\{\frac{1 + (-1)^{n+1}}{2}\right\}_{n=1}^{\infty}$

$a_n = 4 + 3n$ $\quad \{4 + 3n\}_{n=1}^{\infty}$

Advantages?

a) directly compute N^{th} term

b) No ambiguity / No mind reading

1, 2, 4, 8, 16, ...

Web page

#3

3   10   17   24   31   38   . . .

7   7   7   7   7

$a_n = \alpha + \beta n$

$\alpha + \beta = 3$

$\alpha + 2\beta = 10$

$\beta = 7$

$\alpha = -4$

#4

1   2   4   7   11   16   22   25

1   2   3   4   5   6   7

1   1   1   1   1   1

$a_n = \alpha + \beta n + \gamma n^2$

$\alpha + \beta + \gamma = 1$

$\alpha + 2\beta + 4\gamma = 2$

$\alpha + 3\beta + 9\gamma = 4$

$\alpha = 1$

$\beta = -\frac{1}{2}$

$\gamma = \frac{1}{2}$

Arithmetic Sequences

Fixed initial term (a). Fixed difference (d)

$$a_n = a + d(n-1) \quad , n \geq 1$$

#3

Find 22^{th} term arithmetic sequence when

7 is 3rd term , 19th is 5th term

$$\underset{1st}{\underline{-5}},\ \underset{2nd}{\underline{1}},\ \underset{3rd}{\underline{7}},\ \underset{4th}{\underline{13}},\ \underset{5th}{\underline{19}},\ \underset{6th}{\underline{\quad}},\ \ldots$$

$$a_n = a + d(n-1) \quad \begin{cases} n=3 & 7 = a + d\,2 \\ n=5 & 19 = a + d\,4 \end{cases}$$

$\curvearrowright$

$a = -1 \qquad d = 6$

$$a_n = -1 + 6(n-1)$$

Geometric Sequences

Fixed initial term a fixed common ratio r

$a_{n+1} = r\, a_n$

or

$$a_n = a_1 r^{n-1} \qquad \frac{a_{n+1}}{a_n} = r$$

#2

Find ninth term of a geometric sequence with 3rd & 4th terms 6, 4 respect.

$\frac{27}{2}$	9	6	4	___
1	2	3	4	5

$$r = \frac{4}{6} = \frac{2}{3}$$

$$\begin{cases} \dfrac{a_3}{a_2} = \dfrac{6}{a_2} = \dfrac{2}{3} \Rightarrow a_2 = 9 \\[2ex] \dfrac{a_2}{a_1} = \dfrac{2}{3} \Rightarrow a_1 = \dfrac{27}{2} \end{cases}$$

or

$$\begin{cases} \begin{array}{ccc} 6 & 4 & \_\_ \\ \_\_ & \_\_ & \_\_ \\ 1^* & 2^* & 3^* \end{array} \end{cases}$$

$$a_n^* = 6\left(\frac{2}{3}\right)^{n-1} \qquad a_7^* = 6\left(\frac{2}{3}\right)^6$$

a_9

Recursive / Recurrance Sequences

$a_n = \text{relation}(a_1, a_2, \dots, a_{n-1})$

$$\begin{cases} a_n = -\frac{1}{2} a_{n-1}, & n \geq 2 \\ a_1 = 12 \end{cases}$$

one term linear

$$\begin{cases} a_n = a_{n-1} + a_{n-2}, & n \geq 3 \\ a_1 = 1 \\ a_2 = 1 \end{cases}$$

two term linear

Find $a_n = f(n)$??

Backwards substitution

#2 $\quad a_n = -\frac{1}{2} a_{n-1} = \left(-\frac{1}{2}\right)^2 a_{n-2} = \left(-\frac{1}{2}\right)^3 a_{n-3}$

$a_{n-1} = -\frac{1}{2} a_{n-2}$

$a_{n-2} = -\frac{1}{2} a_{n-3}$

$a_n = \dots \left(-\frac{1}{2}\right)^{n-2} a_2 = \left(-\frac{1}{2}\right)^{n-1} a_1$

$a_n = \left(-\frac{1}{2}\right)^{n-1} 12$

$$\begin{cases} a_n = 5a_{n-1} - 6a_{n-2} & n \geq 3 \\ a_1 = 4 \\ a_2 = 11 \end{cases}$$

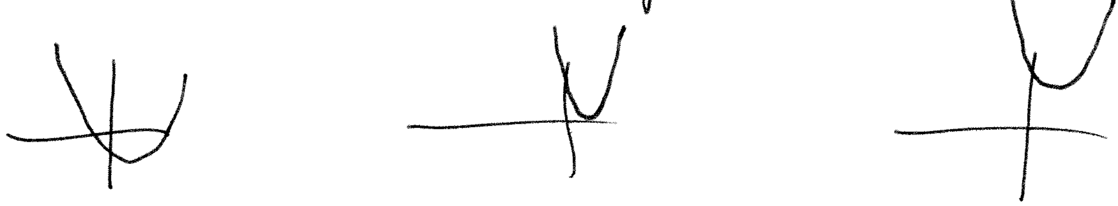
Characteristic Equation

$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

$$\Updownarrow$$

$$s^2 - 5s + 6 = 0 \qquad \text{two roots}$$

two real   one real repeat   two complex



$b^2 - 4ac > 0$ $b^2 - 4ac = 0$ $b^2 - 4ac < 0$

Case I

$s = s_1,\ s_2 = s_2$

$$a_n = \alpha s_1^n + \beta s_2^n$$

$$a_n = \alpha 2^n + \beta 3^n$$

$$\alpha 2 + \beta 3 = 4$$

$$\alpha 4 + \beta 9 = 11$$

$$a_n = \frac{1}{2} 2^n + 3^n$$